DAVRIY KOEFFITSIENTLI SHTURM-LIUVILL OPERATORLARINING KVADRATIK DASTASI UCHUN TESKARI SPEKTRAL MASALA

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Annotatsiya: Hozirgi kunda nochiziqli to'lqinlar nazariyasidagi izlanishlar, jumladan, optik solitonlar telekomunikatsion texnologiyalar sohasida qo'llanila boshladi. Bundan tashqari, fizik sistemalarga ta'sir etuvchi kuchlar faqat vaqtning ma'lum bir davri mobaynida chegaralangan bo'ladi, shuning uchun haqiqiy modellar fazoviy o'zgaruvchilar bo'yicha davriy va deyarli davriy funksiyalar sinfidagi tenglamalarni o'rganishga keltiriladi. Shu munosabat bilan davriy funksiyalar sinfida yuklangan nochiziqli modifitsirlangan Korteveg-de Friz tenglamasini o'rganish maqsadli ilmiy tadqiqotlardan hisoblanadi. Bu paragrafda davriy koeffitsientli Shturm-Liuvill operatorlarining kvadratik dastasi uchun qo`yilgan teskari spektral masalaga oid kerakli ma'lumotlarni keltramiz

Kalit so'zlar: Davriy funksiyalar sinfi, davriy masala, teskari spektral masala, Shturm-Liuvill operatorlarining kvadratik dastasi.

Quyidagi

$$T(\lambda)y \equiv -y'' + q(x)y + 2\lambda p(x)y - \lambda^2 y = 0, \; x \in R \qquad (1.1)$$

Shturm-Liuvill tenglamalarining kvadratik dastasini ko`rib chiqamiz. Bunda $p(x)$ va $q(x)$ haqiqiy, uzluksiz, π davrli funksiyalar, λ esa kompleks parametr.

$c(x,\lambda)$ va $s(x,\lambda)$ orqali (1.1) tenglamaning ushbu $c(0,\lambda)=1$, $c'(0,\lambda)=0$, $s(0,\lambda)=0$, $s'(0,\lambda)=1$ boshlang`ich shartlarni qanoatlantiruvchi yechimlarini belgilaymiz.

Ushbu $\Delta(\lambda) = c(\pi,\lambda) + s'(\pi,\lambda)$ funksiyaga (1.1) tenglamaning Lyapunov funksiyasi yoki Xill diskriminanti deyiladi.

Teorema (Floke). Agar $\Delta^2(\lambda) - 4 \neq 0$ bo`lsa, (1.1) tenglama quyidagi

$$\psi_-(x,\lambda) = \rho_-^{\frac{x}{\pi}} \cdot p_-(x,\lambda), \qquad \psi_+(x,\lambda) = \rho_+^{\frac{x}{\pi}} \cdot p_+(x,\lambda)$$

ko`rinishdagi ikkita chiziqli erkli yechimga ega. Bunda $p_-(x,\lambda)$ va $p_+(x,\lambda)$ funksiyalar $x$ bo`yicha π davrga ega va

$$\rho_\mp = \rho_\mp(\lambda) = \frac{\Delta(\lambda) \pm \sqrt{\Delta^2(\lambda) - 4}}{2};$$

Agar $\Delta(\lambda)=2$ bo`lsa, (1.1) tenglama π davrli yechimga ega;

Agar $\Delta(\lambda)=-2$ bo`lsa, (1.1) tenglama π antidavrli yechimga ega.

Agar $\psi_{\pm}(0,\lambda)=1$ deb olsak, u holda

$$\psi_{\pm}(x,\lambda)=c(x,\lambda)+\frac{s'(\pi,\lambda)-c(\pi,\lambda)\mp\sqrt{\Delta^2(\lambda)-4}}{2s(\pi,\lambda)}s(x,\lambda)$$

bo`ladi.

Floke teoremasidagi $\psi_{\pm}(x,\lambda)$ yechimlarga Floke yechimlari deyiladi.

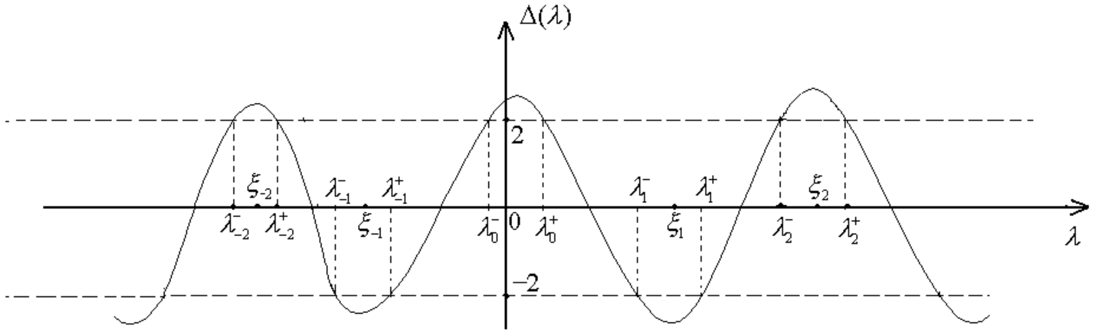
Malumki ([9]), agar $q\in L_2[0,\pi]$ va $p\in W_2^1[0,\pi]$ haqiyqiy, π davrli funksiyalar quyidagi shartlarni qanoatlantirsa: ushbu $y(x)\not\equiv 0$ va $y'(0)\bar{y}(0)-y'(\pi)\bar{y}(\pi)=0$ shartlarni qanoatlantiruvchi $y(x)\in W_2^2[0,\pi]$ funksiyalar uchun

$$\int_0^{\pi}\left\{|y'(x)|^2+q(x)|y(x)|^2\right\}dx>0$$

tengsizlik bajarilsa, u holda (1.1) masalaning spektri quydagi to`plamdan iborat bo`ladi

$$\sigma(T)=\left\{\lambda\in R\,\middle|\,-2\le\Delta(\lambda)\le 2\right\}=R\setminus\bigcup_{n=-\infty}^{\infty}(\lambda_{2n-1},\lambda_{2n}).$$

Bu yerda $(\lambda_{2n-1},\lambda_{2n})$, $n\in Z$ intervallarga lakunalar deyiladi. Nomerlash shunday kiritilganki, bunda $\lambda_{-1}<0<\lambda_0$ qo`sh tengsizlik bajariladi.



(1-rasm)

ξ_n, $n\in Z\setminus\{0\}$ orqali $s(\pi,\lambda)=0$ tenglamaning ildizlarini belgilaymiz. Bu holda ξ_n, $n\in Z\setminus\{0\}$ sonlar, (1.1) tenglama uchun qo`yilgan Dirixle masalasining ($y(0)=y(\pi)=0$) xos qiymatlari bilan ustma-ust tushadi va ushbu $\xi_n\in[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z\setminus\{0\}$ munosabatlar o`rinli bo`ladi.

Ta'rif 1. $\xi_n\in[\lambda_{2n-1},\lambda_{2n}]$, $n\in Z\setminus\{0\}$ sonlarga va ushbu $\sigma_n=sign\{s'(\pi,\xi_n)-c(\pi,\xi_n)\}$, $n\in Z\setminus\{0\}$ ishoralarga (1.1) masalaning spektral parametrlari deyiladi.

Ta'rif 2. ξ_n, σ_n, $n \in Z \setminus \{0\}$ spektral parametrlar va spektrning chegaraviy nuqtalari λ_n, $n \in Z$ ga (1.1) masalaning spektral berilganlari deyiladi.

(1.1) masalaning spektral berilganlarini topish masalasiga to`g`ri spektral masala deyiladi. Aksincha, spektral berilganlar bo`yicha $p(x)$ va $q(x)$ koeffitsientlarni tiklash masalasiga teskari spektral masala deyiladi.

Shturm-Liuvill operatolarining kvadratik dastasini $p(x+\tau)$ va $q(x+\tau)$ koeffitsientlar bilan qarasak, uning spektri haqiqiy parametr τ ga bog`liq bo`lmaydi, ammo spektral parametrlari τ ga bog`liq bo`ladi, ularni $\xi_n(\tau)$, $\sigma_n(\tau)$, $n \in Z \setminus \{0\}$ orqali belgilaymiz. Bu spektral parametrlar quydagi Dubrovin-Trubovits sistemasini qanoatlantiradi:

$$\frac{d\xi_n}{d\tau} = 2(-1)^{n-1} sign(n) \cdot \sigma_n(\tau) \cdot \sqrt{(\xi_n - \lambda_{2n-1})(\lambda_{2n} - \xi_n)} \times$$

$$\times \sqrt{(\xi_n - \lambda_{-1})(\xi_n - \lambda_0) \prod_{k \neq n,0} \frac{(\xi_n - \lambda_{2k-1})(\xi_n - \lambda_{2k})}{(\xi_n - \xi_k)^2}}, \quad n \in Z \setminus \{0\}.$$

Dubrovin-Trubovits sistemasi va ushbu

$$p(\tau) = \frac{\lambda_{-1} + \lambda_0}{2} + \sum_{0 \neq k = -\infty}^{\infty} \left(\frac{\lambda_{2k-1} + \lambda_{2k}}{2} - \xi_k(\tau) \right),$$

$$q(\tau) + 2p^2(\tau) = \frac{(\lambda_{-1})^2 + (\lambda_0)^2}{2} + \sum_{0 \neq k = -\infty}^{\infty} \left(\frac{(\lambda_{2k-1})^2 + (\lambda_{2k})^2}{2} - \xi_k^2(\tau) \right)$$

izlar formulalari birgalikda teskari spektral masalani yechishga imkon beradi. Bundan tashqari ulardan foydalanib, quyidagi teoremalarni isbot qilish mumkin.

Teorema 1. ([12]). Agar (1.1) tenglamaning $p(x)$ va $q(x)$ koeffitsientlari haqiqiy analitik funksiyalar bo`lsa, ya'ni haqiqiy o`qning har bir nuqtasida yaqinlashuvchi darajali qatorga yoyiluvchi bo`lsa, u holda (1.1) tenglamaga mos keluvchi lakunalarning uzunliklari eksponensial ravishda nolga intiladi, ya'ni shunday o`zgarmas musbat sonlar $a > 0$ va $b > 0$ topilib, ushbu $\lambda_{2n} - \lambda_{2n-1} < ae^{-b|n|}$, $n \in Z$ tengsizlik o`rinli bo`ladi.

Teorema 2 ([12]). Agar (1.1) tenglamaga mos keluvchi lakunalar uzunliklari eksponensial ravishda nolga intilsa, u holda uning $p(x)$ va $q(x)$ koeffitsientlari haqiqiy analitik funksiyalar bo`ladi.

Teorema 3. ([13]). Agar $p(x)$ va $q(x)$ koeffitsientlar $\frac{\pi}{2}$ davrga ega bo`lsa, u holda (1.1) tenglamaga mos keluvchi toq nomerli barcha lakunalar yo`qoladi. Aksincha, agar toq nomerli barcha lakunalar yo`qolsa, $p(x)$ va $q(x)$ koeffitsientlar $\frac{\pi}{2}$ davrga ega bo`ladi.

Bu yerda toq nomerli lakuna yo`qoladi deganda uning chetki nuqtalari ustma-ust tushishi nazarda tutiladi. Bu hol antidavriy masalaning xos qiymati ikki karrali bo`lganida sodir bo`ladi.

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