TURG’UNLIK BAHOSI

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Annotatsiya: Bizga ma’lumki xususiy hosilali differensiallar fani hayotimizning ajralmas qismi hisoblanadi. Shu bilan birga yatish mumkinki ular ikki turga bo’linadi. Bular korrekt qo’yilgan va nokorrekt qo’yilgan masalalar deyiladi. Bu mavzuda asosan issiqlik tarqalish tenglamasi ustida ishlaymiz va uning nokorrekt qo’yilgan masalalari haqida suhbat qilamiz.Bundan tashqari teskari qo’yilgan masalalarning turg’unlik bahosi haqida tenglama yordamida hisoblashlar amalga oshiramiz.

Kalit so‘zlar: turg’unlik bahosi, parabolik tip, Koshi-Bunyakovskiy tengsizliklari, nokorrekt masala, issiqlik tarqalish tenglamasi.

Issiqlik tarqalish tenglamasi uchun quyidagi tenglamani qaraymiz:

$$v_t(x,t) = -v_{xx}(x,t),\ \ (x,t)\epsilon\Omega = (0,\pi) * (0,T),\ \ (1)$$

$$v(x,0)=f(x),\qquad x\epsilon[0,\pi],$$

$$v(0,t)=v(\pi,t)=0,\qquad t\epsilon[0,T]. \qquad (2)$$

berilgan f(x) funksiya bo’yicha u(x)=v(x,T) ni toppish masalasini qaraymiz va uning uchun l-turg’unlik bahosini topamiz.

Shu maqsadda quyidagi integralni baholaymiz

$$I=\int_0^T \int_0^\pi \exp(-2st)\,|v_t(x,t) + v_{xx}(x,t)|^2 dxdt,\ \ s > 0,$$

(1) tenglamaga muofiq integral nolga teng.

Integralni v(x,t)=$e^{st}w(x,t)$ *almashtirish yordamida* quydagicha ko’rinishda yozamiz

$$0=I=\int_0^T \int_0^\pi |w_t + sw + w_{xx}|^2\, dxdt =$$

$$= \int_0^T \int_0^\pi |w_t|^2 dxdt + \int_0^T \int_0^\pi |st + w_t|^2 dxdt +$$

$$+2\int_0^T \int_0^\pi (sw_t + w_t w_{xx}) dxdt \geq \int_0^T \int_0^\pi s(\,w^2(x,t))_t dxdt -$$

$$\int_0^T \int_0^\pi 2w_{tx}w_x\, dxdt =$$

$$=\int_0^T \int_0^\pi (s\,w^2 - w_x^2)_t dxdt = \int_0^\pi \big(sw^2(x,t) - w_x^2(x,t)\big)dx\,|_0^T. \qquad (3)$$

Bu tengsizlikni hosil qilishda I integralda ikkita manfiymas qo’shiluvchilarni tashlab yuboramiz, qolgan hadlarini t bo’yicha bo’laklab integrallaymiz. (2) bir jinsli shartlardan va $w_t(0,t) = w_t(\pi,t)$=0 ekanligidan foydalansak, integraldan tashqari ifodalar nolga aylanadi. Natijada (3) dan ushbu tengsizlikni

$$S\int_0^\pi w^2(x,T)dx - \int_0^\pi {w_x}^2(x,T)dx \leq s\int_0^\pi w^2(x,0)dx - -\int_0^\pi {w_x}^2(x,0)dx \leq$$

$$\leq s\int_0^\pi w^2(x,0)dx$$

hosil qilamiz. Quyidagilarni eslab,

w(x,T)=exp(-sT)v(x,T)=exp(-sT)u(x),

w(x,o)=v(x,0)=f(x),

topamiz

exp(-2sT)$\int_0^{\pi}|u(x)|^2dx \leq$ exp(-2sT)$\int_0^{\pi}\left|u_x^{'}\ (x)\right|^2dx + s\int_0^{\pi}|f(x)|^2dx$.

Tengsizlik ikkala tomonini s exp(-2sT)ga bo'lib va $\varepsilon^2 = s^{-1}$ deb hisoblab, kvadrat ildiz olgandan kegin

$$\|u\|_{L_{2(0,\pi)}} \leq \varepsilon\|u_x\|_{L_{2(0,\pi)}} + \exp(T\varepsilon^{-2})\ \|f\|_{L_{2(0,\pi)}}$$

hosil qilamiz.

Shunday qulib, yuqoridagi masala l-korrekt masala ekan, bu yerda

$$l(u) = \|u_x\|_{L_{2(0,\pi)}}.$$

Vaqt bo'yicha yo'nalishini o'zgartiradigan parabolik tipdagi tenglamaga qo'yilgan nokorrekt masala.

Q=(-1<x<1)*(0,T) sohada quyidagi tenglamani qaraymiz

$$\text{signx}u_t(x,t) = -u_{xx}(x,t). \qquad (4)$$

Bu vaqt bo'yicha yo'nalishni o'zgartiradigan parabolik tip tenglamasidir.

(4) tenglamani Q∩{x≠0} sohada quyidagi

a) boshlang'ich

$$u(x,0)=f(x),\ -1\leq x \leq 1;$$

b) chegaraviy

$$u(-1,t)=u(1,t)=0,\quad 0\leq t \leq T;$$

c) tikish

$$u(-0,t)=u(+0,t),\ u_x(-0,t) = u_x(+0,t),\ 0 \leq t \leq T;$$

shartlarni qanoatlantiruvchi yechimni topish masalasini qaraymiz. Bu masala ham korrektmas masaladir. Uning shartli to'g'ri ekanligini isbotlaymiz.

Quyidagi belgilashni kiritamiz

$$\varphi(t) = -\int_{-1}^{1} u_{xx}\, u\, dx = \int_{-1}^{1} u_x^2\, dx.$$

$\varphi(t)$ funksiyani differensallab topamiz

$$\varphi'(t) = -2\int_{-1}^{1} u_{xx}\, u_t\, dx = 2\int_{-1}^{1} u_{xt}\, u_x\, dx,$$

$$\varphi''(t) = -2\int_{-1}^{1} u_{xx}\, u_{tt}\, dx + 2\int_{-1}^{1} u_{xt}^2\,(x,t)dx. \qquad (5)$$

Izoh. Bu yerda u(x,t) funksiyani hisoblashlarda qatnashyotgan barcha mos tartibli hosilalarni mavjud deb faraz qilinadi.

$\varphi''(t)$ ifodani 1-chi integralni ko'rinishini (4) tenglamadan foydalanib o'zgartiramiz

$$-2\int_{-1}^{1} u_{xx}\, u_{tt}\, dx = -2\int_{-1}^{1} \text{sign(x)}u_t(x,t)\,\text{sign(x)}u_{xxt}(x,t)dx=$$

$$=2\int_{-1}^{1} u_{xt}^2\, dx. \qquad (6)$$

(5) va (6) dan topamiz

$$\varphi''(t) = 4\int_{-1}^{1} u_{xt}^2(x,t)\, dx.$$

$\Psi(t) = \ln(\ \ \varphi(t))$ funksiyani kiritib,

$$\Psi''(t) = \frac{\varphi(t)\varphi''(t) - (\varphi'(t))^2}{\varphi^2(t)} =$$

$$= \ 4\,\frac{\int_{-1}^{1} u_{xt}^2(x,t)\, dx \int_{-1}^{1} u^2(x,t)\, dx - \left(\int_{-1}^{1} u_{xt}\,(x,t)u(x,t)dx\right)^2}{\int_{-1}^{1} u^2(x,t)\, dx} \geq 0,$$

Yoki

$$\Psi''(t) \geq 0 \qquad (7)$$

hosil qilamiz.

Bu yerda biz Koshi-Bunyakovskiy tengsizliklardan foydalandik.

(7) kelib chiqadi

$$\Psi(t) \leq \frac{T-t}{T}\Psi(0) + t/T\Psi(T). \qquad (8)$$

(8) potensirlab, topamiz

$$\int_{-1}^{1} u_x^2(x,t)\,dx \leq \quad \{\int_{-1}^{1} u_x^2(x,0)\,dx\}^{(T-t)/T}\{\int_{-1}^{1} u_x^2(x,T)dx\}^{t/T},$$

Yoki

$$\|u_x(x,t)\|_{L_{2(-1,1)}} \leq \|u_x(x,0)\|_{L_{2(-1,1)}}^{(T-t)/T}\|u_x(x,T)\|_{L_{2(-1,1)}}^{t/T}$$

Shunday qilib, yuqoridagi masala shartli korrekt masala bo'lib, korrektlik to'plami quydagicha

$$\text{M}=\left\{x: \middle| \|u_x(x,T)\|_{L_{2(-1,1)}} \leq m\right\},$$

bo'ladi.

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