TRAPETSIYAGA AYRIM DOIR FORMULALARNING ISBOTLARI

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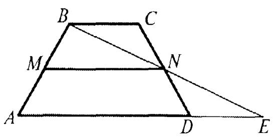
Annotatsiya: Ushbu maqolada trapetsiyaga doir bo'lgan, masalalar yechishda ko'p qo'llaniladigan ayrim formulalarning isbotlarini keltiramiz.

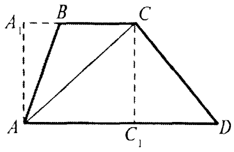
Kalit so'zlar: trapetsiya, o'rta chiziq, balandlik, yuza, diagonal, asos, tengdosh.

1- teorema(Trapetsiya o'rta chizig'i haqida). ***Trapetsiyaning o'rta chizig'i uning asoslariga parallel va asoslari yig'indisining yarmiga teng.***

MN— trapetsiyaning o'rta chizig'i, ya'ni *MA* = *MB* va *DN* = *NC* bo'lsin, u holda $MN \parallel AD$ va $MN = \frac{AD + BC}{2}$ bo'lishini isbotlash talab qilinadi.

I s b o t i. *B* va *N* nuqtalardan *BN* to'g'ri chiziq o'tkazamiz va uni trapetsiya *AD* tomonining davomi bilan *E* nuqtada kesishguncha davom ettiramiz. Natijada ikkita *BNC* va *DNE* uchburchakni hosil qilamiz (1- chizma).





1- chizma 2- chizma

Shartga ko'ra *CN* = *ND* vertikal burchaklar sifatida $\angle BNC = \angle DNE$ bo'lganligidan hamda ikkita parallel *BC* va *DE* to'g'ri chiziq va ularni *CD* to'g'ri chiziq bilan kesganda hosil bo'lgan burchaklar sifatida $\angle BCN = \angle NDE$ bo'lganligidan, $\Delta BNC = \Delta NDE$. Uchburchaklarning tengligidan, *BN=NE* va *BC=DE* bo'ladi.

Demak, *MN* kesma ΔABE ning o'rta chizig'idir. Uchburchak o'rta chizig'ining xossasiga ko'ra $MN \parallel AE$ va $MN = \frac{AE}{2} = \frac{AD + DE}{2} = \frac{AD + BC}{2}$. Teorema isbotlandi.

Natija: Trapetsiya diagonallarining o'rtalarini tutashtiruvchi kesmaning uzunligi uning asoslari uzunliklari ayirmasining yarmiga teng.

2- teorema(Trapetsiyaning yuzi haqida). ***Trapetsiyaning yuzi uning asoslari yig'indisining yarmi bilan balandligining ko'paytmasiga teng.***

Agar *ABCD* trapetsiyaning asoslari *AD* = *a*, *BC* = *b*, balandligi $CC_1 = AA_1 = h$ bo'lsa (2- chizma), trapetsiyaning yuzi

$S = \frac{a+b}{2} \cdot h$ formula bo'yicha hisoblanishini isbotlash talab qilinadi.

I s b o t i. Trapetsiyaning *AC* diagonalini o'tkazamiz, natijada trapetsiya ikkita, *ACD* va *ABC* uchburchakka ajraladi. A va C nuqtalardan $AA_1 \perp BC$ va $CC_1 \perp AD$ balandliklar o'tkazamiz. $AD||BC$ bo'lganligidan, $CC_1 = AA_1 = h$ bo'ladi. Shu sababli, *ACD* va *ABC* uchburchaklarning yuzlari

$$S_{\Delta ACD} = \frac{1}{2}ah; \qquad S_{\Delta ABC} = \frac{1}{2}bh;$$

formulalar bo'yicha hisoblanadi. Trapetsiyaning yuzi esa

$S = S_{\Delta ACD} + S_{\Delta ABC} = \frac{1}{2}ah + \frac{1}{2}bh = \frac{a+b}{2}h$ bo'ladi. Teorema isbotlandi.

3 - t e o r e m a(Ixtiyoriy qavariq to'rtburchakning yuzi haqida) . ***Qavariq to'rtburchakning yuzi uning diagonallari ko 'paytmasining yarmi bilan ular orasidagi burchak sinusining ko'paytmasiga teng.***

ABCD qavariq to'rtburchakda $AC=d_1$, $BD = d_2$ diagonallari va ular orasidagi $\angle COD = \alpha$ burchak ma'lum bo'lsin. U holda to'rtburchakning yuzi

$S = \frac{1}{2}d_1 d_2 \sin\alpha$ formula bo'yicha hisoblanishini isbotlash kerak.

I s b o t i. Qavariq *ABCD* to'rtburchakning *AC*, *BD* diagonallari (3-chizma) to'rtburchakni to'rtta *AOB, BOC, COD, AOD* uchburchakka bo'ladi. Ma'lumki, $\angle AOB = \angle COD = \alpha$, $\angle BOC = \angle AOD = 180° - \alpha$, u holda

$$S_{\Delta AOB} = \frac{1}{2} \cdot AO \cdot BO \cdot \sin\alpha, \quad S_{\Delta BOC} = \frac{1}{2} \cdot BO \cdot CO \cdot \sin(180^\circ - \alpha) = \frac{1}{2} \cdot BO \cdot CO \cdot \sin\alpha,$$

$$S_{\Delta COD} = \frac{1}{2} \cdot CO \cdot OD \cdot \sin\alpha, \quad S_{\Delta AOD} = \frac{1}{2} \cdot AO \cdot OD \cdot \sin(180^\circ - \alpha) = \frac{1}{2} \cdot AO \cdot OD \cdot \sin\alpha$$

Demak, *ABCD* to'rtburchakning yuzi

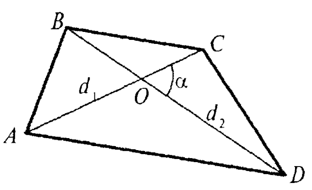
$$S_{ABCD} = S_{\Delta AOB} + S_{\Delta BOC} + S_{\Delta COD} + S_{\Delta AOD} =$$

$$= \frac{1}{2} \cdot AO \cdot BO \cdot \sin\alpha + \frac{1}{2} \cdot BO \cdot CO \cdot \sin\alpha + \frac{1}{2} \cdot CO \cdot OD \cdot \sin\alpha + \frac{1}{2} \cdot AO \cdot OD \cdot \sin\alpha =$$

$$= \frac{1}{2}BO(AO + CO)\sin\alpha + \frac{1}{2}OD(CO + AO)\sin\alpha = \frac{1}{2}BO \cdot AC \cdot \sin\alpha + \frac{1}{2}OD \cdot AC \cdot \sin\alpha =$$

$$= \frac{1}{2} \cdot AC \cdot (BO + DO)\sin\alpha = \frac{1}{2} \cdot AC \cdot BD \cdot \sin\alpha$$

bo'ladi.



3- chizma

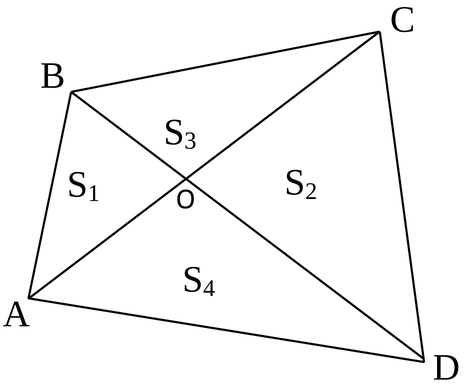
Shartga ko'ra $AC=d_1$, $BD = d_2$ bo'lganligidan talab qilingan

$$S = \frac{1}{2} d_1 d_2 \sin\alpha$$

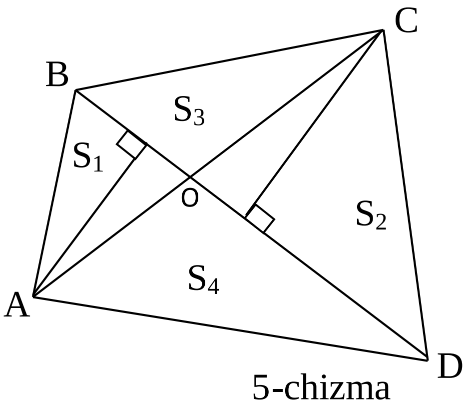
munosabatni olamiz.

Faraz qilaylik, *a, b, c, d*— to'rtburchakning tomonlari va $P = \frac{a+b+c+d}{2}$ uning yarim penmetri bo`lsin.

4-teorema. **Ixtiyoriy qavariq ABCD to'rtburchakning diagonallari O nuqtada kesishishidan hosil bo'lgan AOB, BOC, COD va AOD uchburchaklar (4-chizma) ning yuzalari uchun quyidagi tenglik o'rinli :** $\boldsymbol{S_1 \cdot S_2 = S_3 \cdot S_4}$



4-chizma



5-chizma

Isboti: AC yoki BD diagonallarning biriga h_1 va h_2 balandliklar tushirilgan bo'lsin(5-chizma), hamda $BO=m$, $OD=n$ belgilashlarni kiritib olaylik . U holda uchburchaklar yuzlari uchun quyidagilarni yoza olamiz:

$$S_1 = \frac{m \cdot h_1}{2}, \quad S_2 = \frac{n \cdot h_2}{2}, \quad S_3 = \frac{m \cdot h_2}{2}, \quad S_4 = \frac{n \cdot h_1}{2}$$

Yuza uchun olingan tengliklardan quyidagi nisbatlarni hosil qilamiz:

$$\frac{S_1}{S_3} = \frac{h_1}{h_2} \quad \text{va} \quad \frac{S_4}{S_2} = \frac{h_1}{h_2}$$

Oxirgi nisbatlarning o'ng tomonlari tengligidan ularning chap tomonlarini ham tenglasak bizga isbotlanishi kerak bo'lgan tenglik kelib chiqadi. Teorema isbotlandi.

To'rtburchakning aynan trapetsiya turida S_1 va S_2 yuzalar teng bo'ladi, chunki ABC va BCD uchburchaklarning BC asosi umumiy va balandliklari teng. $S_1 = S_2$ ekanligi va 4-teoremani e'tiborga oladigan bo'lsak, quyidagi tenglikka ega bo'lamiz: $\boldsymbol{S_1 = S_2 = \sqrt{S_3 \cdot S_4}}$

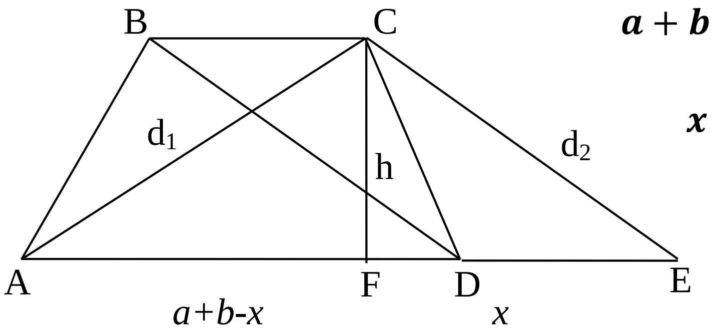
Oxirgi tenglikni e'tiborga olib, trapetsiyaning yuzi uchun

$$\boldsymbol{S = \left(\sqrt{S_3} + \sqrt{S_4}\right)^2}$$

formulaga ega bo'lamiz.

5-teorema(Trapetsiyaning yuzi haqida): **Diagonallari d_1 va d_2, balandligi h bo'lgan trapetsiyaning yuzi** $\boldsymbol{S = \frac{\sqrt{d_1^2 - h^2} + \sqrt{d_2^2 - h^2}}{2} \cdot h}$ **formula bilan hisoblanadi.**

Isboti: Formulani isbotlash uchun trapetsiyaning *BD* diagonalini parallel holda *C* uchiga ko'chiramiz va hosil qilingan *ACE* uchburchakning *CF* balandligini tushirib, *AF=a+b-x*, *FE=x* belgilashlarni kiritamiz. Shundan keyin ACF va CEF uchburchaklarga Pifagor teoremasini qo'llaymiz,



$$a + b - x = \sqrt{d_1^2 - h^2}$$

$$x = \sqrt{d_2^2 - h^2}$$

5-chizma

Hosil qilingan tengliklarni qo'shib asoslar yig'indisini hosil qilamiz va trapetsiya yuzi formulasini qo'llab isbotlanishi talab qilingan formulani hosil qilamiz:

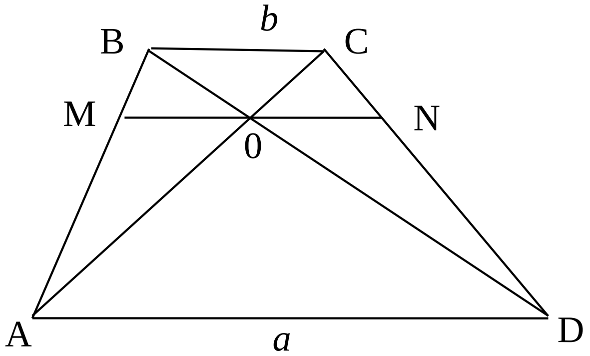
$$a + b = \sqrt{d_1^2 - h^2} + \sqrt{d_2^2 - h^2}$$

$$S = \frac{a+b}{2} = \frac{\sqrt{d_1^2-h^2}+\sqrt{d_2^2-h^2}}{2} \cdot h$$ Teorema isbotlandi.

6-teorema: **Asoslari *a* va *b* bo'lgan ixtiyoriy trapetsiyaning diagonallari kesishgan nuqtadan asoslariga parallel holda o'tgan kesmaning uzunligi $x = \frac{2ab}{a+b}$ ga teng bo'ladi.**

Isboti: AOD va BOC uchburchaklar(6-chizma) o'xshashligidan

$\frac{AO}{OC} = \frac{a}{b}$ (*)



Shuningdek, ABC va AMO uchburchaklar o'xshashligidan

$\frac{AC}{AO} = \frac{b}{MO} \rightarrow \frac{AO+OC}{AO} = \frac{b}{MO} \rightarrow$

$1 + \frac{OC}{AO} = \frac{b}{MO}$ (*) ga ko'ra $1 + \frac{b}{a} = \frac{b}{MO}$

6-chizma

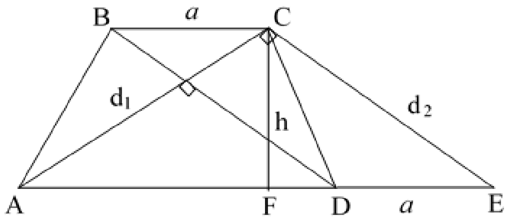
$MO = \frac{ab}{a+b}$ ga ega bo'lamiz. MO va ON teng ekanligidan $MN = 2 \cdot MO = \frac{2ab}{a+b}$ kelib chiqadi. Teorema isbotlandi.

7-teorema(Trapetsiyaning balandligi haqida)**: Diagonallari o'zaro perpendikular bo'lgan ixtiyoriy trapetsiyaning balandligi uning diagonallari orqali quyidagicha topiladi:** $\boldsymbol{h}=\frac{\boldsymbol{d_1}\cdot\boldsymbol{d_2}}{\sqrt{\boldsymbol{d_1^2}+\boldsymbol{d_2^2}}}$

Isboti: Trapetsiyaning *BD* diagonalini parallel ravishda *C* uchiga ko'chiramiz(7-chizma). Natijada *ACE* to'g'ri burchakli uchburchak hosil bo'ladi.

ACE uchburchakning katetlari d_1 va d_2 bo'lib, AE gipotenuza esa Pifagor teoremasiga ko'ra $AE=\sqrt{d_1^2+d_2^2}$ ga teng.

To'g'ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligi uchun o'rinli bo'lgan $h=\frac{ab}{c}$



7-chizma

formulani qo'llasak, $\boldsymbol{h}=\frac{\boldsymbol{d_1}\cdot\boldsymbol{d_2}}{\sqrt{\boldsymbol{d_1^2}+\boldsymbol{d_2^2}}}$ ekanligi kelib chiqadi. Teorema isbotlandi.

8-teorema(Trapetsiyaning yuzi haqida): **Diagonallari o'zaro perpendikular bo'lgan teng yonli trapetsiyaning yuzi uning balandligi(yoki o'rta chizig'i)ning kvadratiga teng bo'ladi.**

Isboti: Teoremani isbotlashda 7-chizmadan foydalanamiz . *ACE* to'g'ri burchakli uchburchak teng yonli bo'ladi, chunki shartga ko'ra trapetsiya teng yonli va shuning uchun uning diagonallari o'zaro teng bo'ladi. Uchburchakning AE gipotenuzasi $a+b$ ga teng va gipotenuzaga tushirilgan balandlik bir vaqtda mediana ham bo'ladi, demak $h=\frac{a+b}{2}$. Bundan esa $S=h^2$ ekanligi kelib chiqadi.

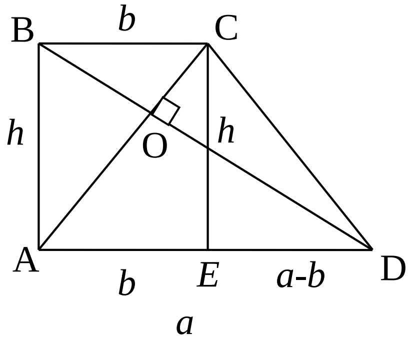
Teorema isbotlandi.

8-teorema: **To'g'ri burchakli trapetsiyaning asoslari** ***a*** **va** ***b*** **bo'lib, diagonallari o'zaro perpendikulyar bo'lsa, uning balandligi** $h=\sqrt{ab}$ **ga teng bo'ladi**.

Isboti:

$\Delta BOC\sim\Delta AOD$ dan $\frac{AO}{OC}=\frac{DO}{OB}=\frac{a}{b}\rightarrow$

$AO=\frac{a}{b}\cdot OC;\quad DO=\frac{a}{b}\cdot OB$



To'g'ri burchakli *ABD* va *ACE* uchburchaklardan

18-chizma

$BD^2=h^2+a^2\ \rightarrow\ BO+OD=\sqrt{h^2+a^2}\ \rightarrow BO+\frac{a}{b}BO=\sqrt{h^2+a^2}\quad\rightarrow$

$AC^2=h^2+b^2\ \rightarrow\ AO+OC=\sqrt{h^2+b^2}\ \rightarrow\frac{a}{b}OC+OC=\sqrt{h^2+b^2}\quad\rightarrow$

$$BO = \frac{b}{a+b}\sqrt{h^2+a^2} \quad va \quad OC = \frac{b}{a+b}\sqrt{h^2+b^2}$$

ΔBOC dan $BO^2 + OC^2 = BC^2 \rightarrow \frac{b^2}{(a+b)^2}(a^2+h^2) + \frac{b^2}{(a+b)^2}(b^2+h^2) = b^2 \rightarrow$

$(a^2+h^2) + (b^2+h^2) = (a+b)^2 \rightarrow h = \sqrt{ab}$

Xulosa. Xulosa qilib aytishimiz mumkinki, o'quvchi bitta formulani isboti bilan o'rganishi orqali, shu formulani qo'llash mumkin bo'lgan minglab masalalarni yechishni tushunib oladi.

Foydalanilgan adabiyotlar ro'yxati

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